McMaster University
Department of Electrical and Computer Engineering
Photonics Research Laboratory

PROBLEM-INDEPENDENT ENHANCEMENT OF PML ABC FOR TIME DOMAIN TECHNIQUES IN ELECTRODYNAMICS

Yotka Rickard, Natalia Georgieva

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Outline

- Motivation and Objectives
- Background, Challenges and Solutions
- Implementation and Validation
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Classification of Well Known Numerical Techniques for Full Wave EM Field Analysis

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- Finite Element Methods
- Mode Matching Methods
- Moment Methods

Time Domain Methods
- FDTD Methods
- TLM Methods
- IE Methods

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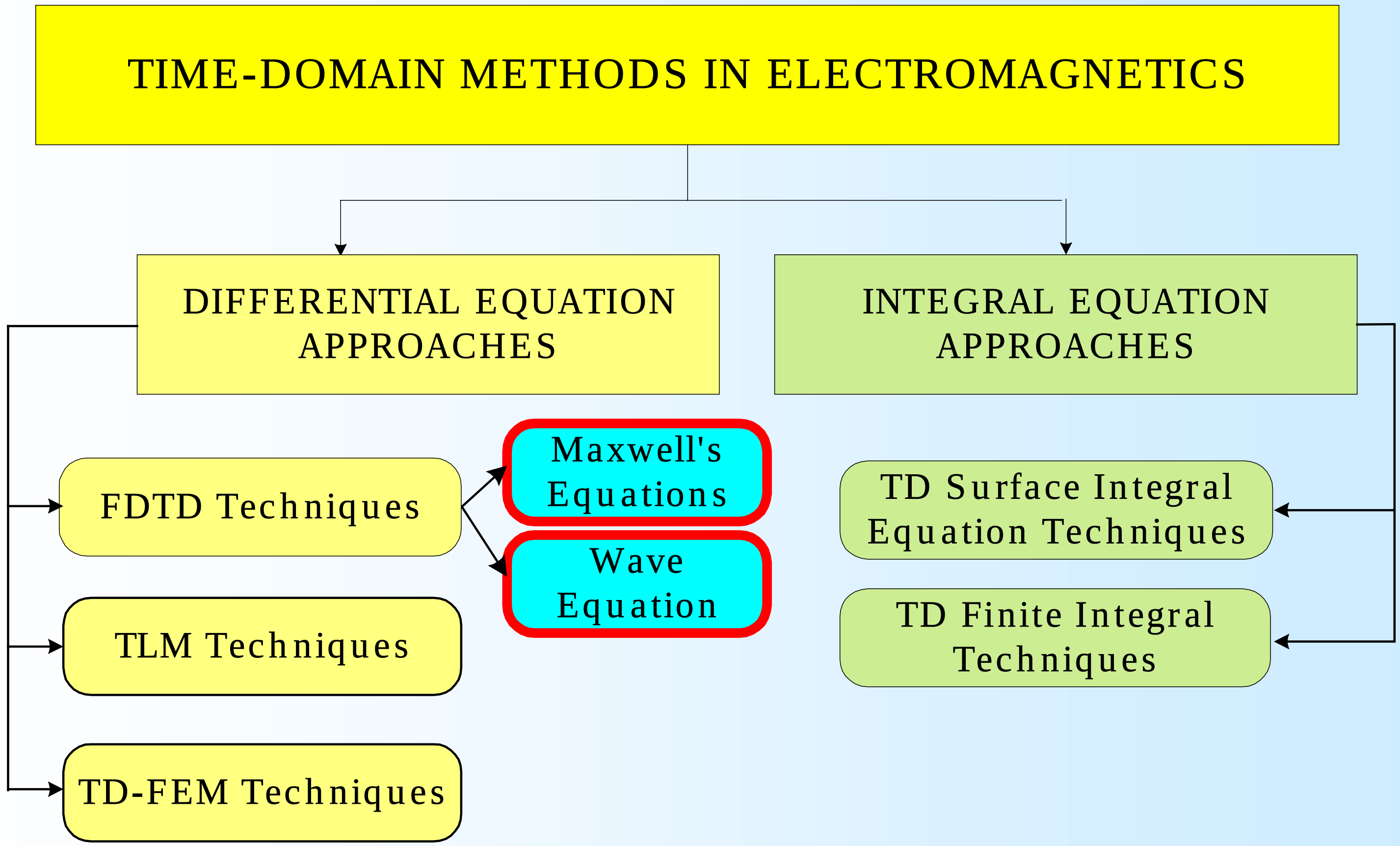
Classification of Time-Domain Numerical Techniques for Full Wave EM Analysis



- **New wave-equation techniques in the time domain** emerging:

1. Vector Potential Technique — N. Georgieva and E. Yamashita (1998), *IEEE Trans. MTT*, vol. 46, No. 4, pp. 404-410.
2. Wave Potential Technique — N. Georgieva and Y. Rickard (2000), *IEEE MTT-S Int. Symposium Digest*, vol. 2, pp. 1129-1132.

- **Reliable, low-reflection ABCs necessary** for both:

1. Open Problems
2. Guided-Wave Problems

- **Practical applications of the ABCs** in:

1. Microwave Electromagnetics
2. Photonics
3. Any physical problem described by the wave equation or Maxwell's equations in the time domain and requiring reflection-free boundaries

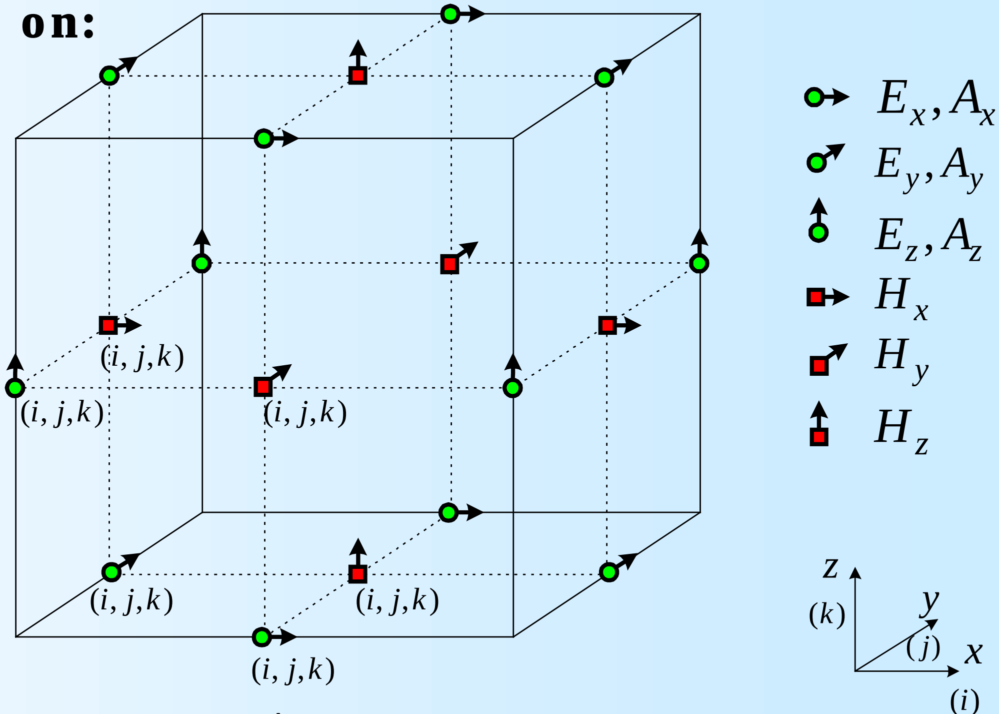
- **Objective:** **Problem-Independent Enhancement of PML ABC for Time-Domain Techniques in Electrodynamics**

- **Time-Domain Methods Based on:**

1. Maxwell's curl equations:

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} - \sigma_m \vec{H} + \vec{J}_m^{\,i}$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E} + \vec{J}^{\,i}$$



$\Rightarrow$ **FDTD** (Finite-Difference Time-Domain)

K.S. Yee (1966) , *IEEE Trans. AP*, vol. 14, 1966, pp. 302-307

2. The wave equation in the time domain: $\Rightarrow$ **WETD**

$$\frac{\partial^2 \vec{A}}{\partial t^2} + \left(\frac{\sigma}{\varepsilon} + \frac{\sigma_m}{\mu}\right)\frac{\partial \vec{A}}{\partial t} + \frac{\sigma\sigma_m}{\varepsilon\mu}\vec{A} - \frac{1}{\mu\varepsilon}\nabla^2 \vec{A} - \nabla\left(\frac{1}{\mu\varepsilon}\right)\nabla\cdot\vec{A} - \Phi\nabla\left(\frac{\sigma}{\varepsilon}\right) = \frac{\vec{J}^{\,i}}{\varepsilon}$$

N. Georgieva and Y. Rickard (2000), *IEEE MTT-S Int. Symp. Digest*, vol. 2, pp. 1129-1132

- **Berenger's Perfectly Matched Layer (PML) ABC** for the FDTD solution to Maxwell's equations: J.P. Berenger (1994), *J. of Comp. Physics*, vol. 114, pp. 185-200.

1. Split the field components (introducing a new degree of freedom in Yee's FDTD algorithm):

$$\varepsilon_0 \frac{\partial E_x}{\partial t} + \sigma E_x = \frac{\partial H_z}{\partial y}$$
$$\varepsilon_0 \frac{\partial E_y}{\partial t} + \sigma E_y = \frac{\partial H_z}{\partial x}$$
$$\mu_0 \frac{\partial H_z}{\partial t} + \sigma^m H_z = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}$$

$$\Rightarrow \begin{cases} \varepsilon_0 \dfrac{\partial E_x}{\partial t} + \sigma_y E_x = \dfrac{\partial \left(H_{zx} + H_{zy}\right)}{\partial y} \\ \varepsilon_0 \dfrac{\partial E_y}{\partial t} + \sigma_x E_y = \dfrac{\partial \left(H_{zx} + H_{zy}\right)}{\partial x} \\ \mu_0 \dfrac{\partial H_{zx}}{\partial t} + \sigma_x^m H_{zx} = -\dfrac{\partial E_y}{\partial x} \\ \mu_0 \dfrac{\partial H_{zy}}{\partial t} + \sigma_y^m H_{zy} = -\dfrac{\partial E_x}{\partial y} \end{cases}$$

2. Ensure reflectionless transmission by:
   - Tangential wave numbers matching and
   - Normal impedance matching.

$$\Rightarrow \frac{\sigma_i}{\varepsilon_0} = \frac{\sigma_i^m}{\mu_0}; \; i = x, \; y, \; z$$

3. Gradually increase the direction-dependent PML conductivity.

- **From Berenger's PML ABC to Chen's MPML ABC –** introduction of PML loss factor (a new degree of freedom): B. Chen, D.G. Fang and B.H. Zhou (1995), *IEEE MGWL*, vol.5, No. 11, pp. 399-401.

$$\mu_0 \kappa_y \frac{\partial H_{xy}}{\partial t} + \sigma_y^m H_{xy} = -\frac{\partial \left(E_{zx} + E_{zy} \right)}{\partial y}$$

$$\mu_0 \kappa_z \frac{\partial H_{xz}}{\partial t} + \sigma_z^m H_{xz} = \frac{\partial \left(E_{yx} + E_{yz} \right)}{\partial z}$$

$$\varepsilon_0 \alpha_y \frac{\partial E_{xy}}{\partial t} + \sigma_y E_{xy} = \frac{\partial \left(H_{zx} + H_{zy} \right)}{\partial y}$$

$$\varepsilon_0 \alpha_z \frac{\partial E_{xz}}{\partial t} + \sigma_z E_{xz} = -\frac{\partial \left(H_{yx} + H_{yz} \right)}{\partial z}$$

PML loss factor for evanescent wave attenuation

PML conductivity for propagating wave attenuation

- **Challenges**:

1. Developing a PML ABC for the wave equation:
   - for time-domain applications in 3 dimensions,
   - for general lossy inhomogeneous media.
2. Developing new PML variable profiles suitable for the wave equation applications.
3. Improving the PML absorber's performance for WETD and FDTD.

- **Solutions**:

1. Use the stretched coordinate approach by mapping the WE into the frequency domain and define auxiliary variables to map it back into the time domain.
2. Introduce a new degree of freedom in the definition of the PML variable profiles – allow growth at different exponent rates.
3. Introduce new types of lossy termination walls for the PML absorbers.

PML for WETD – The Stretched Coordinate Approach

1. W.C. Chew and W.H. Weedon (1994), *MOTL*, vol.7, No. 13, pp.599-604.

$$\nabla_s = \hat{x}\frac{1}{s_x}\frac{\partial}{\partial x} + \hat{y}\frac{1}{s_y}\frac{\partial}{\partial y} + \hat{z}\frac{1}{s_z}\frac{\partial}{\partial z}$$

2. D. Zhou, W.P. Huang, C.L. Xu and D.G. Fang (2001), *IEEE PTL*, vol.13, No.5, pp. 454-456.

Extend D.Zhou's approach to 3 dimensions:

$$\nabla^2\vec{A} - \mu\varepsilon\frac{\partial^2\vec{A}}{\partial t^2} - (\sigma_m\varepsilon + \mu\sigma)\frac{\partial\vec{A}}{\partial t} - \sigma\sigma_m\vec{A} = -\mu\vec{J}$$

$$\nabla_s^2 = \frac{1}{s_x}\frac{\partial}{\partial x}\left(\frac{1}{s_x}\frac{\partial}{\partial x}\right) + \frac{1}{s_y}\frac{\partial}{\partial y}\left(\frac{1}{s_y}\frac{\partial}{\partial y}\right) + \frac{1}{s_z}\frac{\partial}{\partial z}\left(\frac{1}{s_z}\frac{\partial}{\partial z}\right);$$

$$s_\xi = \alpha_\xi + \sigma_\xi/(i\omega\varepsilon)$$
$$\xi = x, y, z$$

$$\nabla_s^2\tilde{\vec{A}} - \mu\varepsilon(i\omega)^2\tilde{\vec{A}} - (\mu\sigma + \varepsilon\sigma_m)i\omega\tilde{\vec{A}} - \sigma\sigma_m\tilde{\vec{A}} = -\mu\tilde{\vec{J}}$$

PML for WETD – The Stretched Coordinate Approach, cont'd

Introduce auxiliary variables:

$$i\omega\tilde{\tilde{\vec{X}}}_1 = \frac{1}{s_x}\frac{\partial\tilde{\tilde{\vec{A}}}}{\partial x}; \quad i\omega\tilde{\tilde{\vec{X}}}_2 = \frac{1}{s_x}\frac{\partial\left(i\omega\tilde{\tilde{\vec{X}}}_1\right)}{\partial x}$$

$$i\omega\tilde{\tilde{\vec{Y}}}_1 = \frac{1}{s_y}\frac{\partial\tilde{\tilde{\vec{A}}}}{\partial y}; \quad i\omega\tilde{\tilde{\vec{Y}}}_2 = \frac{1}{s_y}\frac{\partial\left(i\omega\tilde{\tilde{\vec{Y}}}_1\right)}{\partial y}$$

$$i\omega\tilde{\tilde{\vec{Z}}}_1 = \frac{1}{s_z}\frac{\partial\tilde{\tilde{\vec{A}}}}{\partial z}; \quad i\omega\tilde{\tilde{\vec{Z}}}_2 = \frac{1}{s_z}\frac{\partial\left(i\omega\tilde{\tilde{\vec{Z}}}_1\right)}{\partial z}$$

$\Rightarrow$

$$\alpha_x\frac{\partial\vec{X}_1}{\partial t} + \frac{\sigma_x}{\varepsilon}\vec{X}_1 = \frac{\partial\vec{A}}{\partial x}; \quad \alpha_x\frac{\partial\vec{X}_2}{\partial t} + \frac{\sigma_x}{\varepsilon}\vec{X}_2 = \frac{\partial^2\vec{X}_1}{\partial x\partial t}$$

$$\alpha_y\frac{\partial\vec{Y}_1}{\partial t} + \frac{\sigma_y}{\varepsilon}\vec{Y}_1 = \frac{\partial\vec{A}}{\partial y}; \quad \alpha_y\frac{\partial\vec{Y}_2}{\partial t} + \frac{\sigma_y}{\varepsilon}\vec{Y}_2 = \frac{\partial^2\vec{Y}_1}{\partial y\partial t}$$

$$\alpha_z\frac{\partial\vec{Z}_1}{\partial t} + \frac{\sigma_z}{\varepsilon}\vec{Z}_1 = \frac{\partial\vec{A}}{\partial z}; \quad \alpha_z\frac{\partial\vec{Z}_2}{\partial t} + \frac{\sigma_z}{\varepsilon}\vec{Z}_2 = \frac{\partial^2\vec{Z}_1}{\partial z\partial t}$$

$$\mu\varepsilon(i\omega)^2\tilde{\tilde{\vec{A}}} + (\mu\sigma + \varepsilon\sigma_m)i\omega\tilde{\tilde{\vec{A}}} + \sigma\sigma_m\tilde{\tilde{\vec{A}}} = i\omega\tilde{\tilde{\vec{X}}}_2 + i\omega\tilde{\tilde{\vec{Y}}}_2 + i\omega\tilde{\tilde{\vec{Z}}}_2 + \mu\tilde{\tilde{\vec{J}}}$$

$$\Rightarrow \quad \mu\varepsilon\frac{\partial^2\vec{A}}{\partial t^2} + (\varepsilon\sigma_m + \mu\sigma)\frac{\partial\vec{A}}{\partial t} + \sigma\sigma_m\vec{A} = \frac{\partial\vec{X}_2}{\partial t} + \frac{\partial\vec{Y}_2}{\partial t} + \frac{\partial\vec{Z}_2}{\partial t} + \mu\vec{J}$$

- **Problem-Independent Enhancement of the PML ABC:**

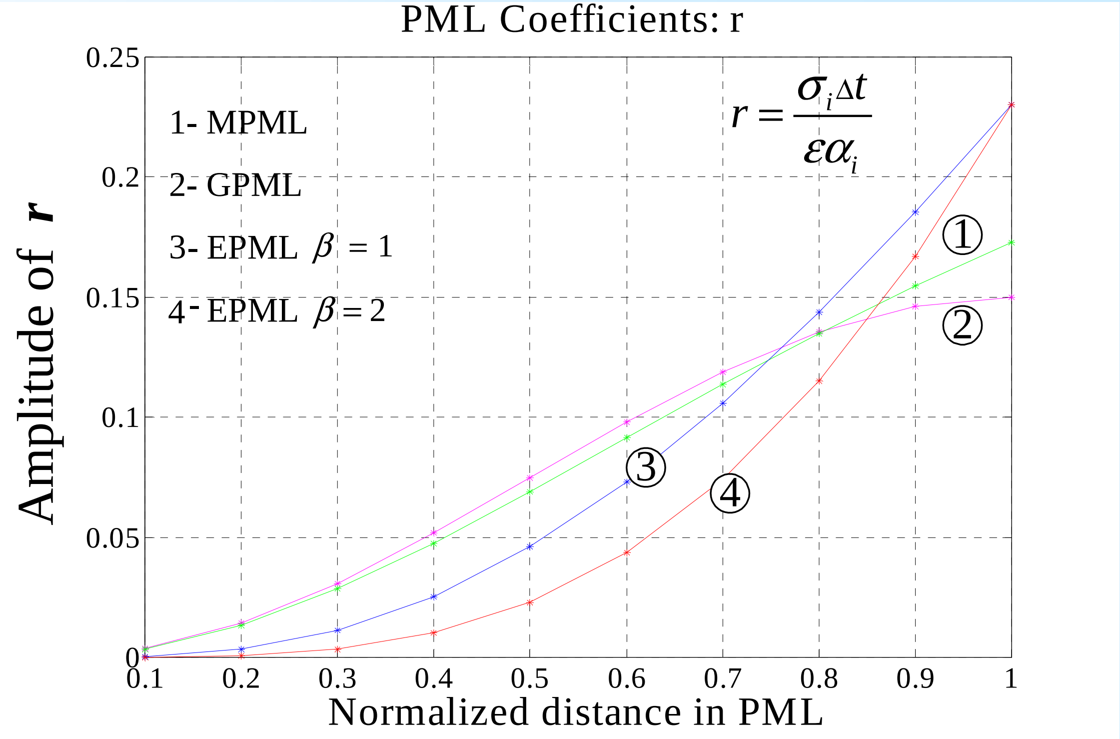
1. Introduction of a new degree of freedom β in the definition of the PML variables:

$$\sigma_i(\rho) = \sigma_{max}\left(\frac{\rho}{\delta_i}\right)^{n+\beta};$$

$$\beta \in (0,3]$$

$$\alpha_i(\rho) = 1 + \varepsilon_{max}\left(\frac{\rho}{\delta_i}\right)^{n}$$

$$r = \frac{\sigma_i \Delta t}{\varepsilon \alpha_i}$$



$$\rho / \delta_i$$

- **Problem-Independent Enhancement of the PML ABC, cont'd:**

2. Introduction of new types of lossy PML termination walls:

<u>The WETD in factored form:</u>

$$(L^2 - \mu\varepsilon\nabla^2)\{\vec{A}\} = 0 \quad \Rightarrow \quad \text{1-D version :} \quad L^+L^-\{\vec{A}\} = 0$$

where

$$L^{\pm} = L \pm v_\xi \partial / \partial\xi$$

$$L = \frac{\partial}{\partial t} + \frac{1}{\tau}$$

$$\Downarrow$$

$$\frac{\sigma}{\varepsilon} = \frac{\sigma_m}{\mu} = \frac{1}{\tau}$$

$$\frac{1}{v_\xi}\left(\frac{\partial \vec{A}}{\partial t} + \frac{\vec{A}}{\tau}\right) + \frac{\partial \vec{A}}{\partial \xi} = 0$$

(One-way WE in lossy medium)

Types of Lossy PML Termination Walls:

A. Lossy one-way wave equation

Proposed by: C.M. Rappaport (1996), *IEEE Trans. on Magnetics*, vol. 32, no. 3, pp. 968-974, May 1996.

$$\frac{1}{v_\xi}\left(\frac{\partial\vec{A}}{\partial t}+\frac{\vec{A}}{\tau}\right)+\frac{\partial\vec{A}}{\partial\xi}=0$$

B. Lossy version of Mur's second order ABC

$$\frac{\partial^2\vec{A}}{\partial t^2}=-\frac{2}{\tau}\frac{\partial\vec{A}}{\partial t}-v\frac{\partial^2\vec{A}}{\partial\xi\partial t}-\frac{v}{\tau}\frac{\partial\vec{A}}{\partial x}-\frac{1}{\tau^2}\vec{A}+\frac{v^2}{2}\left(\frac{\partial^2\vec{A}}{\partial\eta^2}+\frac{\partial^2\vec{A}}{\partial\zeta^2}\right)$$

C. Lossy version of Litva's second order DBC

$$\frac{\partial^2\vec{A}}{\partial t^2}=-\frac{2}{\tau}\frac{\partial\vec{A}}{\partial t}-\left(v_{1\xi}+v_{2\xi}\right)\frac{\partial^2\vec{A}}{\partial\xi\partial t}-\frac{\left(v_{1\xi}+v_{2\xi}\right)}{\tau}\frac{\partial\vec{A}}{\partial x}-\frac{1}{\tau^2}\vec{A}-v_{1\xi}v_{2\xi}\frac{\partial^2\vec{A}}{\partial\xi^2}$$

Implementation and Validation

- **Examples of EPML for FDTD**

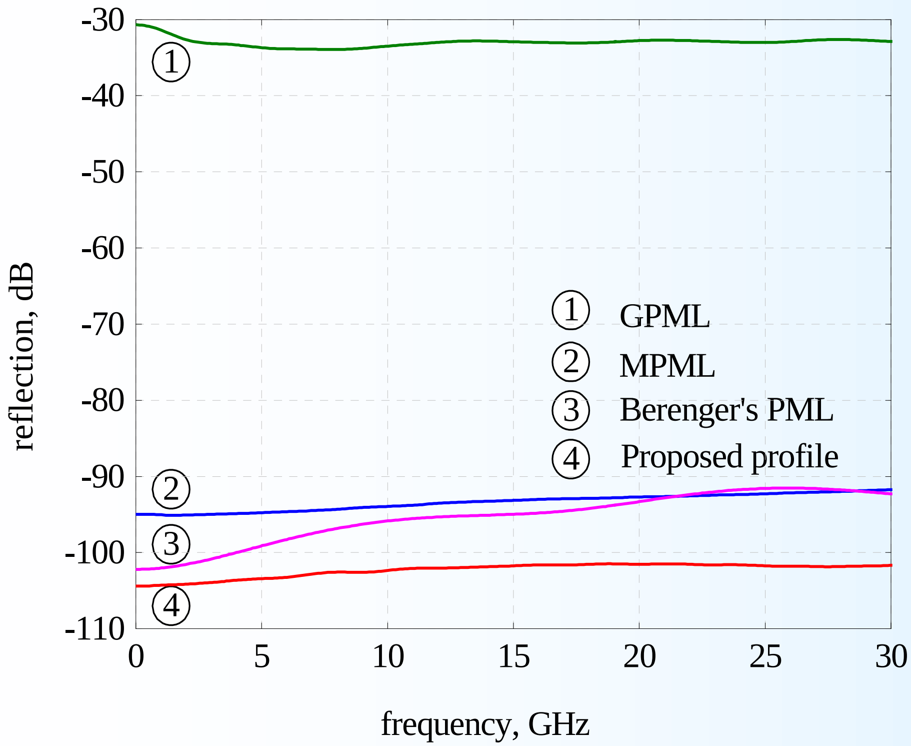
1. Dipole in open space
2. Hollow waveguide
3. Microstrip line
4. Optical waveguide with 2-layer AR coating

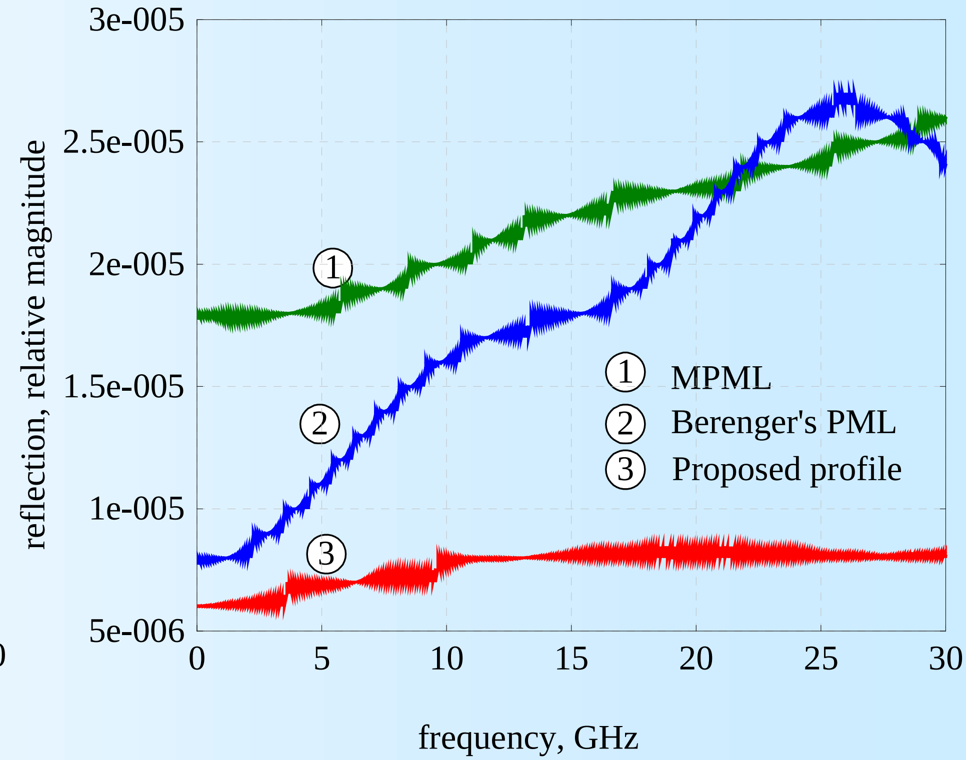
- **Examples of EPML for WETD**

1. Dipole in open space
2. Hollow waveguide
3. Partially filled with dielectric waveguide
4. Optical waveguide with 2-layer AR coating

Reflections:
$$R_{\mathrm{dB}} = 20\log_{10}\left|\frac{\mathfrak{F}\{E_z^{refl}\}}{\mathfrak{F}\{E_z^{inc}\}}\right|$$

Microstrip line

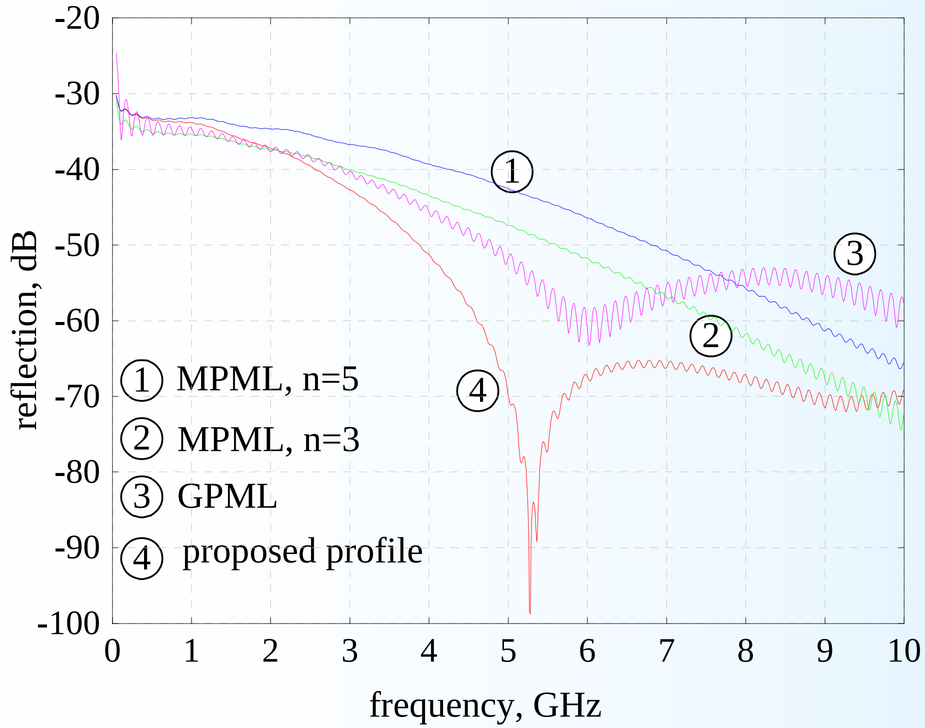




$$N_{\text{PML}} = 8,\ \ R_0 = 10^{-3},\ \ \varepsilon_{\max} = 1,\ \ n = 3$$

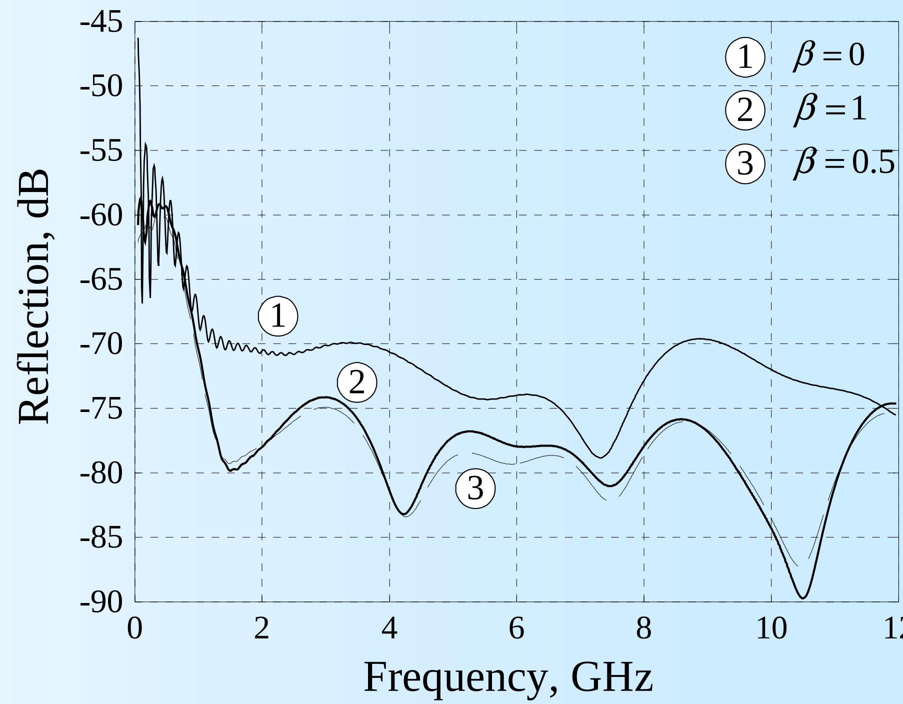
EPML for FDTD

Infinitesimal dipole in open space



$$N_{\mathrm{PML}} = 16,\ R_0 = 10^{-3},\ \varepsilon_{\max} = 3,\ n = 3$$

EPML for WETD

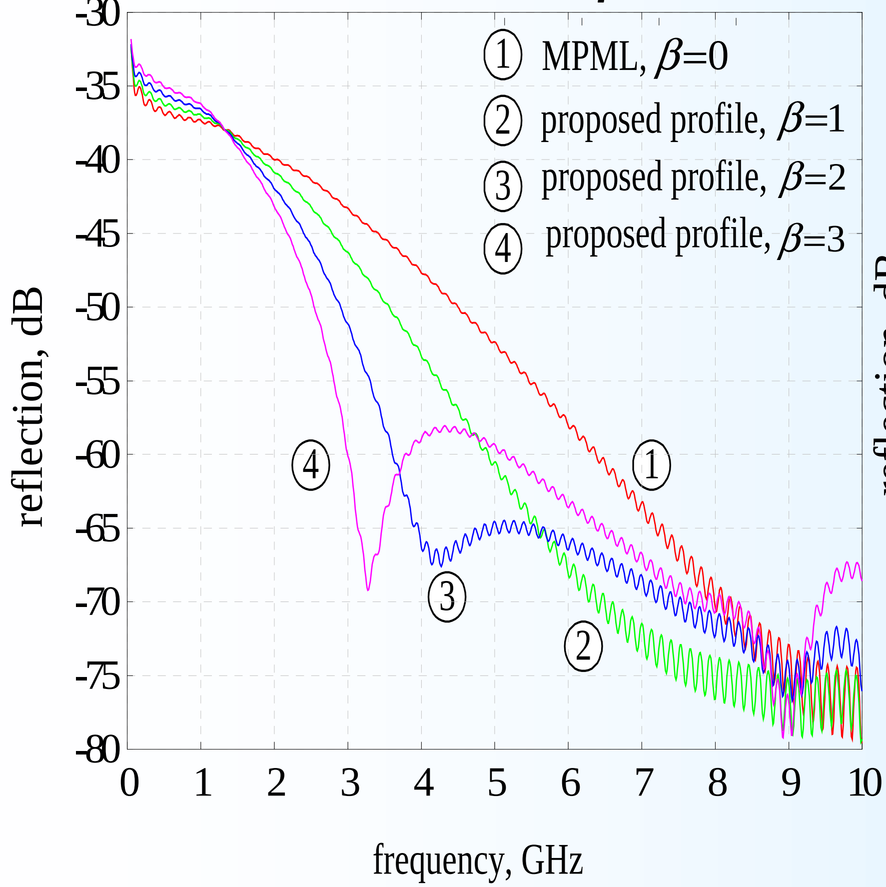


$$N_{\mathrm{PML}} = 10,\ R_0 = 10^{-4},\ \varepsilon_{\max} = 0,\ n = 2$$

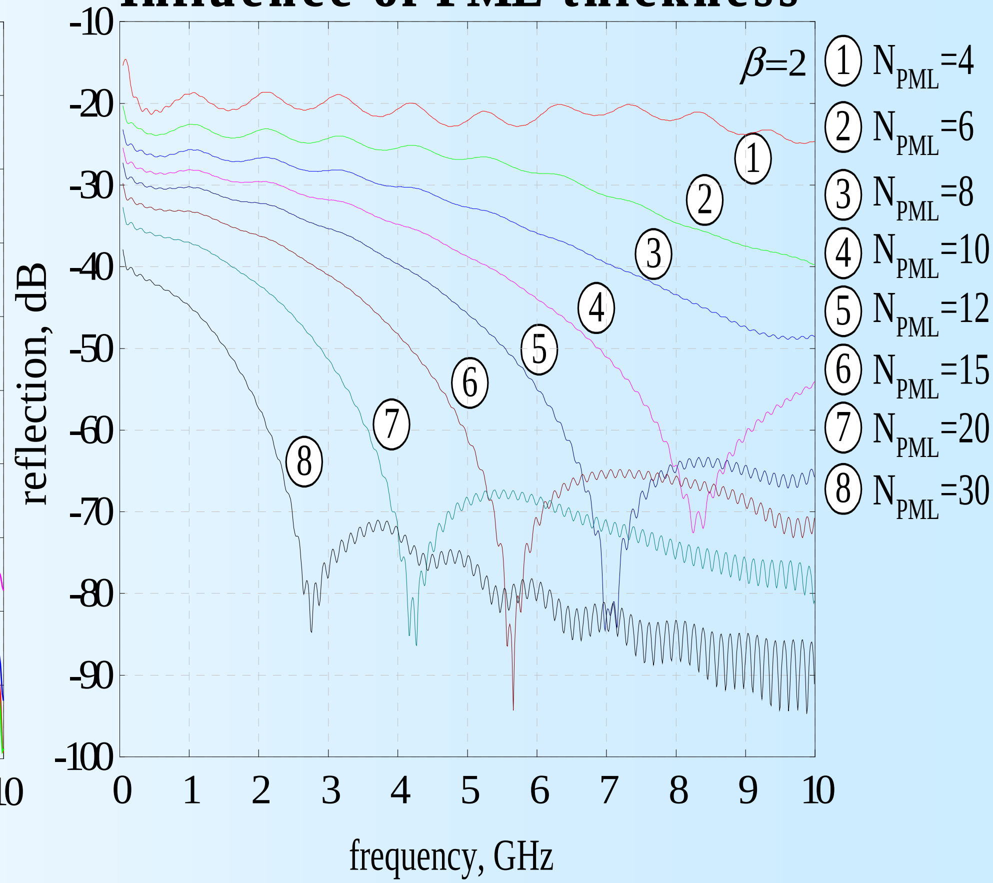
EPML for FDTD

Infinitesimal dipole in open space

Influence of β

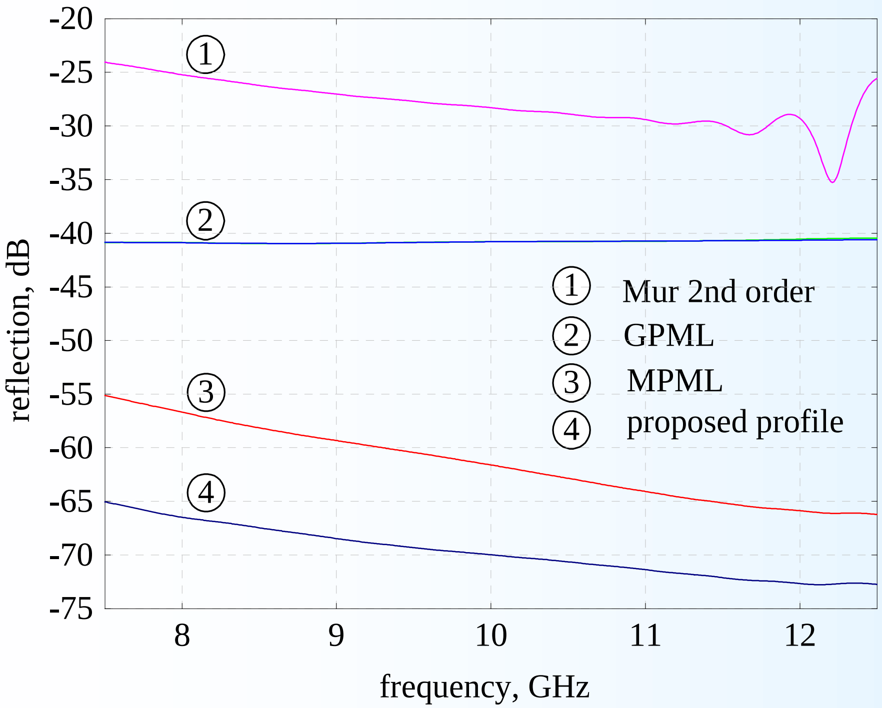


Influence of PML thickness

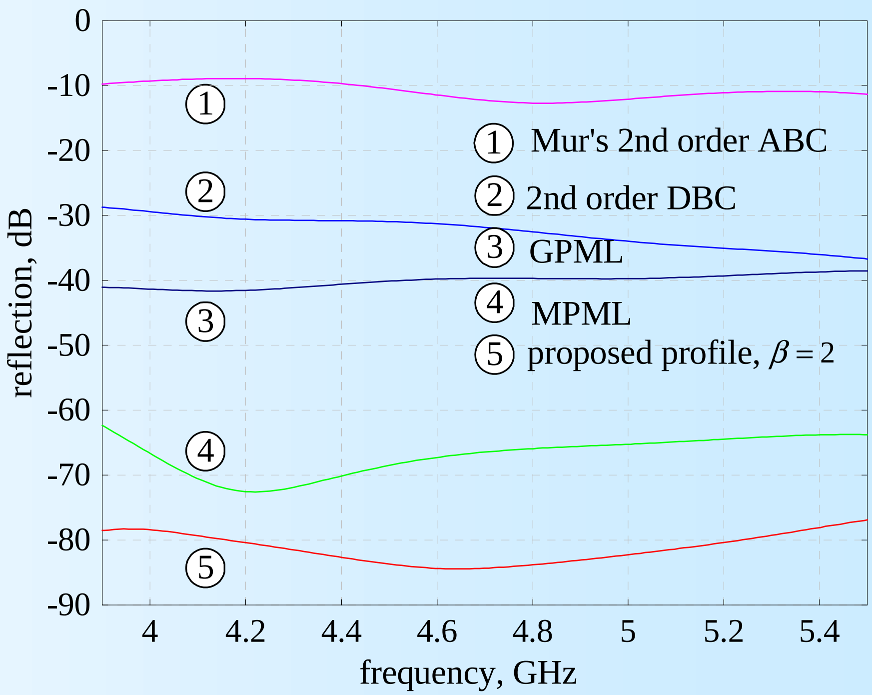


EPML for WETD

Hollow waveguide



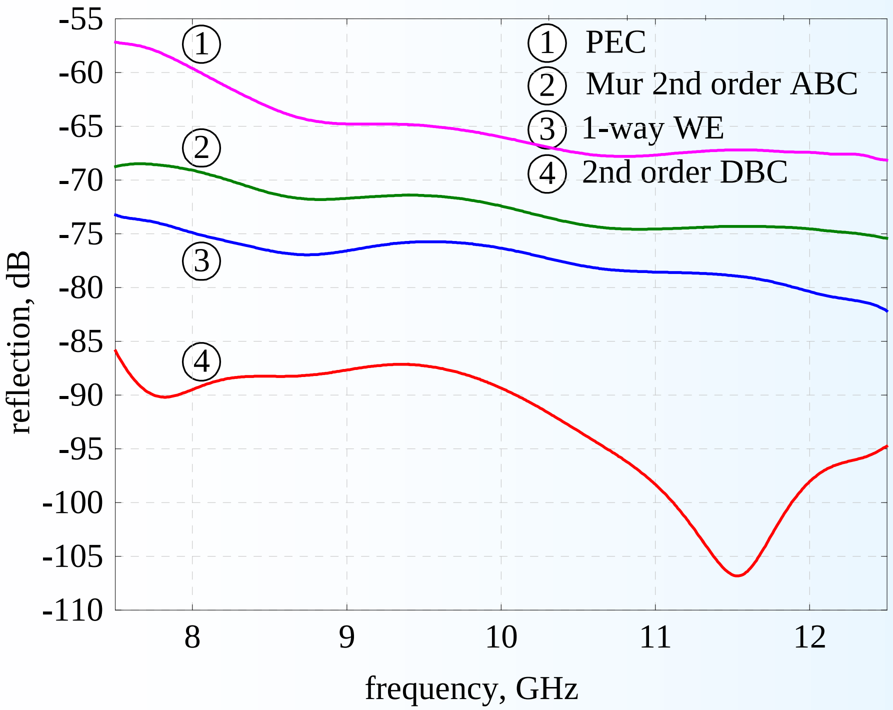
Waveguide partially filled with dielectric



EPML for WETD

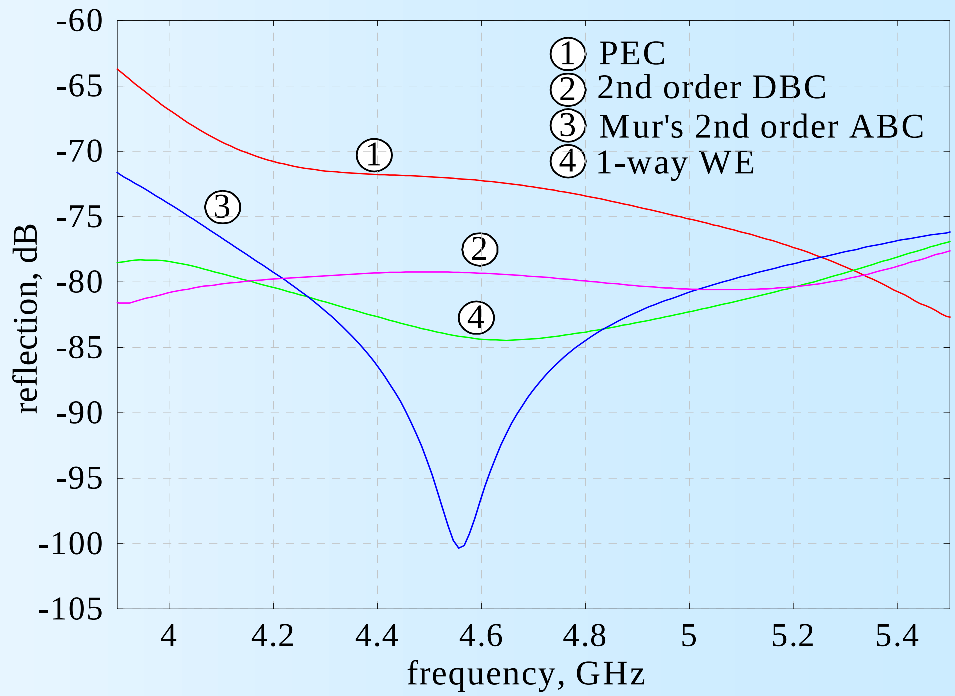
Influence of the Lossy PML Termination Wall

Hollow Waveguide



EPML for FDTD

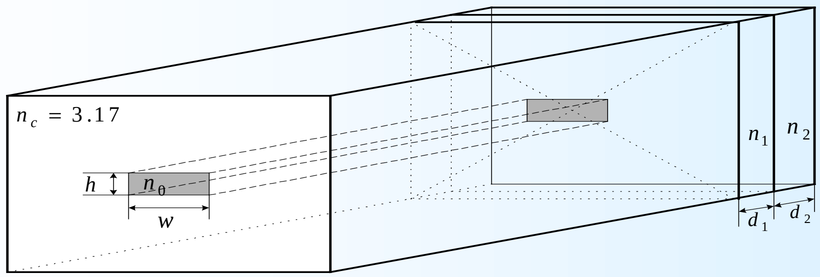
Waveguide Partially Filled with Dielectric



EPML for WETD

Lossy one-way WE – proposed by C.M. Rappaport (1996), *IEEE Trans. on Magnetics*, vol. 32, no. 3, pp. 968-974, May 1996.

Optical Waveguide with a Two-Layer Antireflection Coating

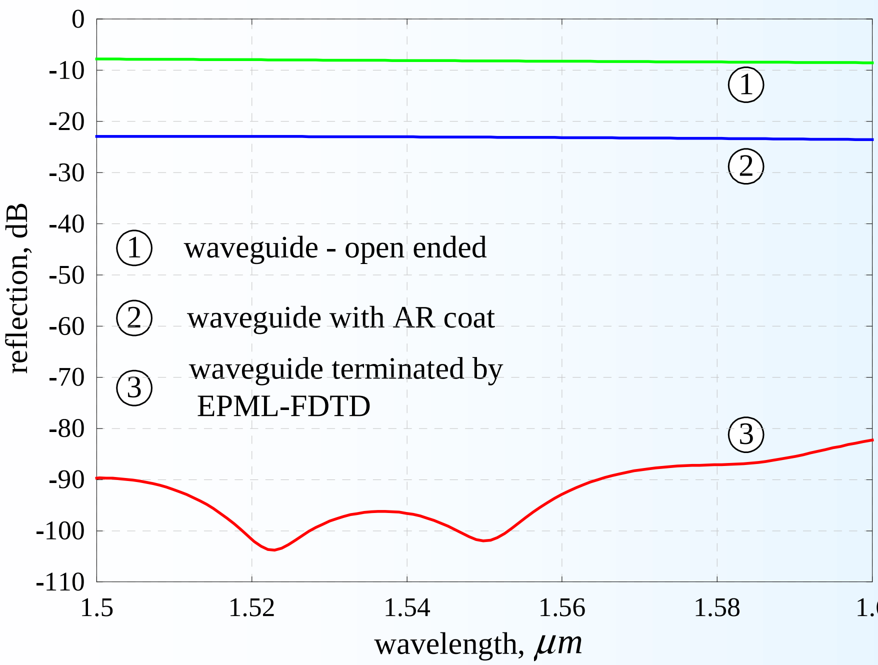


$d_1 = 0.166\,\mu m$ $n_1 = 2.33$

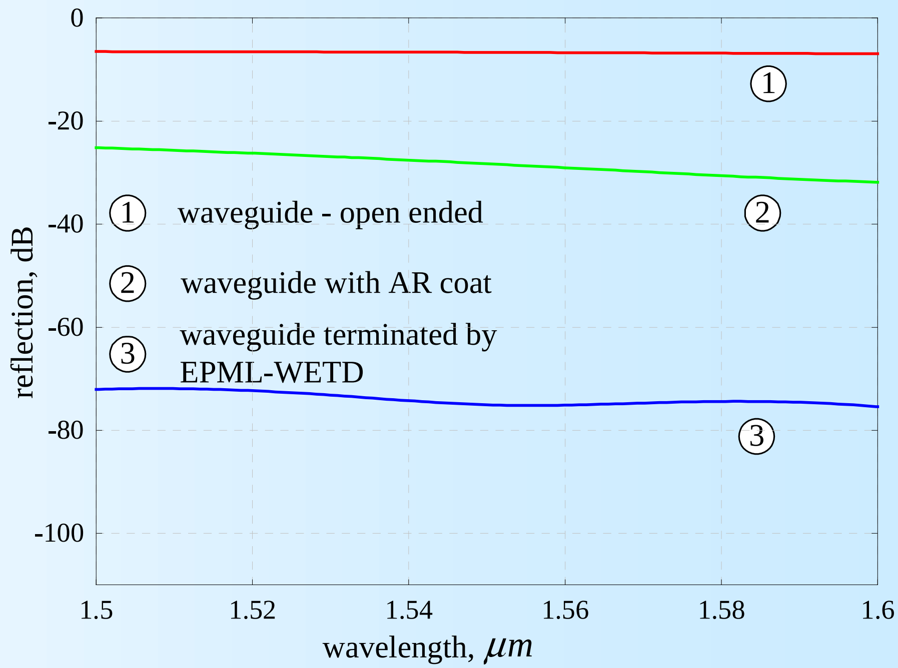
$d_2 = 0.267\,\mu m$ $n_2 = 1.45$

$w = 1.6\,\mu m$ $n_0 = 3.38$

$h = 0.1684\,\mu m$



EPML for FDTD



EPML for WETD

- **Conclusion**

Problem-independent enhancement of PML ABC for the classical Yee's FDTD method and for the WETD method is achieved by the combination of:

1. Introduction of new degree of freedom for PML variable profiles in the spatially polynomial scaling

2. Introduction of modified lossy PML termination walls

- **Suggestions for further development:**

1. Optimization based on the *combined* influence of all EPML parameters on its performance, including the permittivity of the medium, the user-defined reflection coefficient and the 3rd order discretization error.

2. Generalization of the EPML ABC for anisotropic and dispersive media.

3. Application of the EPML ABC to non-linear media.

- **List of Publications Related to the Topic:**

1. Y. Rickard, "Improved Absorbing Boundary Conditions for Time-Domain Methods in Electromagnetics", Ph.D. Thesis, McMaster University, Feb. 2002.
2. Y. Rickard, N. Georgieva and W.-P. Huang, "Application and optimization of PML ABC for the 3-D wave equation in the time domain", in print, *IEEE Trans. on Antennas and Propagation*, Dec 2002.
3. Y. Rickard, N. Georgieva and W.-P. Huang, "A perfectly matched layer for the 3-D wave equation in the time domain", in print, *IEEE Microwave and Wireless Components Letters*, May 2002.
4. Y. Rickard, N. Georgieva and H. Tam, "ABCs for the adjoint problems in the design sensitivity analysis with the FDTD method", submitted to *IEEE Trans. On Microwave Theory and Techniques*.
5. N. Georgieva and Y. Rickard, "The application of the wave potential functions to the analysis of transient electromagnetic fields", *IEEE MTT-S Int. Symposium Digest* (Boston, Massachusetts), June 2000, vol. 2, pp. 1129-1132.
6. N. Georgieva and Y. Rickard, "Time domain modeling of electromagnetic field propagation via wave potentials," *XXVIth General Assembly of the International Union of Radio Science (URSI)*, August 1999, Toronto, Abstracts Digest pp. 178.